Structures hors équilibre à grande échelle en plasmas naturels : limites de la MHD (2)

or: "The importance of not being Maxwellian"

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ENS (11 Février 2008)

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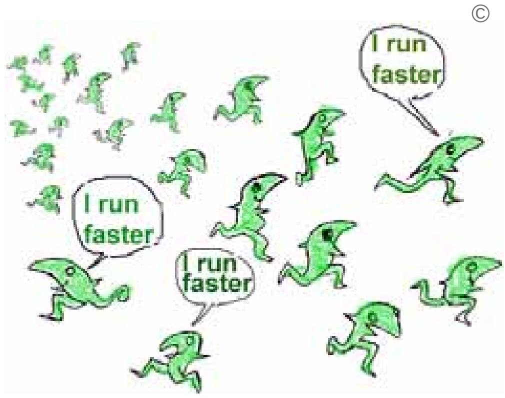
What fluid (MHD) models are doing:

Basically: replace eqs. of motion of N >> 1 particles



Planet. Space Sci. **49** 247 (2001)

by **a few local** differential eqs.on **a few** macroscopic parameters (n,V,T,..)

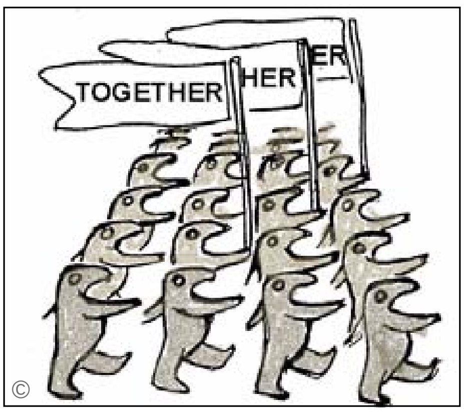
- continuity equation
- momentum equation
- energy equation
- ~~.....~~

Infinite hierarchy of coupled differential equations

☞ **Closing the hierarchy:**

- assume nearly (bi-)Maxwellian $(l \ll H)$
- OR ad-hoc **heat transfer,** or ..

Each species: one fluid with one T (possibly $T_\parallel \neq T_\perp$)



Niveaux de description

Cas classique :
v << c $\qquad$ $d \sim n^{-1/3} >> h/mv$

- **N particules individuelles**
  - ***état*** : ***r***, ***v*** pour chaque particule à *t*
  - ***évolution*** : éq. mouv. de chaque particule
  - *mécanique analytique/constantes du mouv.*

moyennes

description impossible si N trop grand (quoique …)

- **fonction de distribution**
  - ***état*** : *f*(***r***, ***v***) à *t* (proba. de trouver particules avec position ***r***, vitesse ***v***)
  - ***évolution*** : éq. d'évolution de *f*(***r***, ***v***)
  - *mécanique statistique*

en général f défini par une infinité de paramètres

moyennes

- **fluide**
  - ***état*** : **quelques** grandeurs macroscopiques ***n***, ***V***, ***P***, ... de l'élément de fluide en ***r***, *t*
  - ***évolution*** : éqs. de conservation (nombre de particules, qté de mouv., énergie, ..
  - $\Rightarrow$ *équations différentielles locales*

défini par petit nombre de paramètres

Problème: Fermeture de la hiérarchie: Equation d'énergie

Cas le plus simple: Pression isotrope *P*, $\frac{\partial}{\partial t} = 0$

- $P \propto \rho^{\gamma}$ $\Rightarrow$ (Bernouilli) $\qquad \boldsymbol{V}\cdot\nabla\left(\frac{V^2}{2} + \frac{\gamma}{\gamma-1}\frac{k_B T}{m} + \Phi_G\right) = 0$

ou:

- Flux de chaleur $Q \neq 0$: $\rho\boldsymbol{V}\cdot\nabla\left(\frac{V^2}{2} + \frac{\gamma}{\gamma-1}\frac{k_B T}{m} + \Phi_G\right) = -\nabla\cdot\boldsymbol{Q}$

$\boldsymbol{Q} = -\mathcal{K}\nabla T$ si $l_{pm} << L \sim T / \nabla T$

conductivité thermique classique $\qquad \mathcal{K} \sim (3/2)\, n\, k_B\, v_{the}\, l_{pm}$

$$\frac{\text{conduction}}{\text{advection}} \sim \frac{\kappa T/L^2}{nVk_B T/L} \sim \frac{v_{the} l_{pm}}{VL}$$

électrons

$\Rightarrow \nabla.\boldsymbol{Q}$ domine

MAIS : $\mathcal{K} \neq$ valeur classique dès que $l_{pm} / L > 10^{-3}$

(Scudder & Olbert 1983, Shoub 1983, Canullo & al.1996, Dorelli & Scudder, 1999, 2003, Pantellini & Landi 2001)

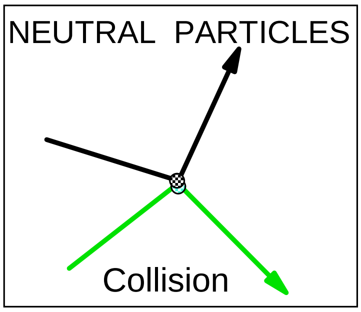
Car :
- Q déterminé par électrons suprathermiques
- électrons suprathermiques non collisionnels ($l >> l_{pm}$)

Collisions : Difference between plasmas and neutral gases

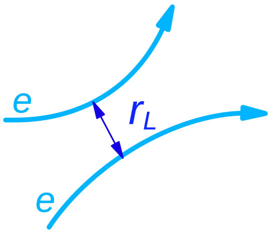
Neutral particles

collisional cross-section ~ "physical section"

in order of magnitude



Charged particles : Coulomb interactions



For a (relative) speed v, a significant perturbation requires distance r_L such that: $\frac{e^2}{4\pi\varepsilon_0 r_L} \sim m v^2$

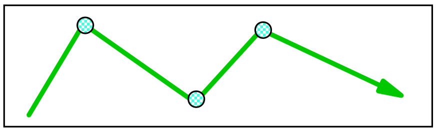
Coulomb energy

cross section : $\sigma_C \sim r_L^2$ $\Rightarrow$ free path $l(v) \propto v^4$

(x lnΛ) $\propto 1/v^4$ $\Rightarrow$ if v x 3 then $l(v)$ x 100 !

fast particules are collisionless, even when most particles (the core of the velocity distribution) are collisional

- Trajectoire d'une particule neutre →

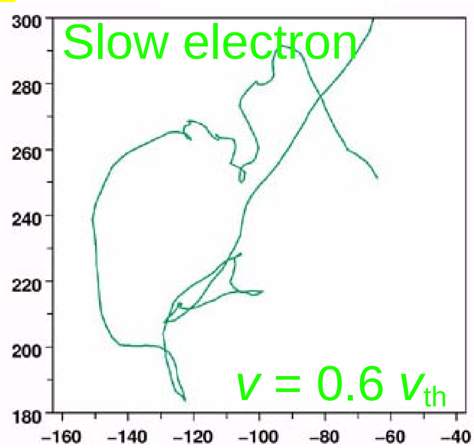


Trajectoire d'un électron en plasma gazeux

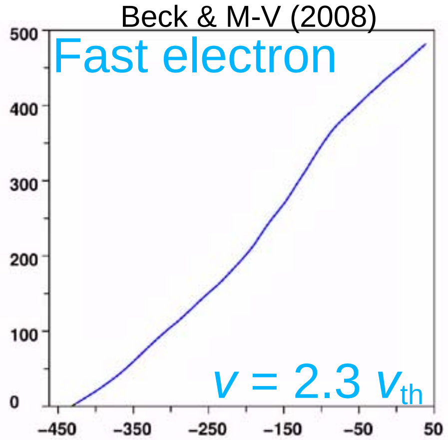
$r_L << \langle r \rangle \sim n^{-1/3}$ $\Rightarrow$ la plupart des interactions produisent petites perturbations

$\Gamma \equiv \frac{e^2}{4\pi\varepsilon_0 \langle r \rangle k_B T} \ll 1$

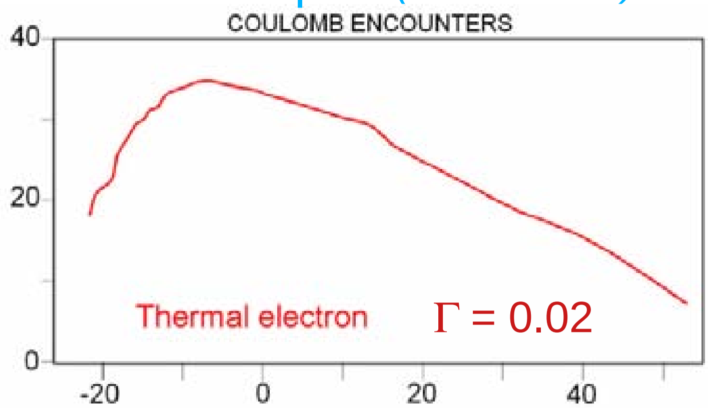
$L_D/\langle r \rangle = (4\pi\Gamma)^{-1/2} \gg 1$ $\Rightarrow$ 1 particule subit beaucoup d'interactions simultanément



Beck & M-V (2008)



simulation n-corps *(Arnaud Beck)*



unité: distance moyenne entre particules

☛ *fast particules are collisionless*

☛ *various acceleration processes*

fast particules are not Maxwellian

⇒ fonction de distribution ≠ Maxwellian

Température hors équilibre local ?

Par ex. : Température cinétique : $\frac{3}{2}k_B T = \int d^3v\,(mv^2/2)\,f(\boldsymbol{v})$

dans repère $\langle \boldsymbol{v} \rangle = 0$ normalisation $\int d^3v\, f(\boldsymbol{v}) = 1$

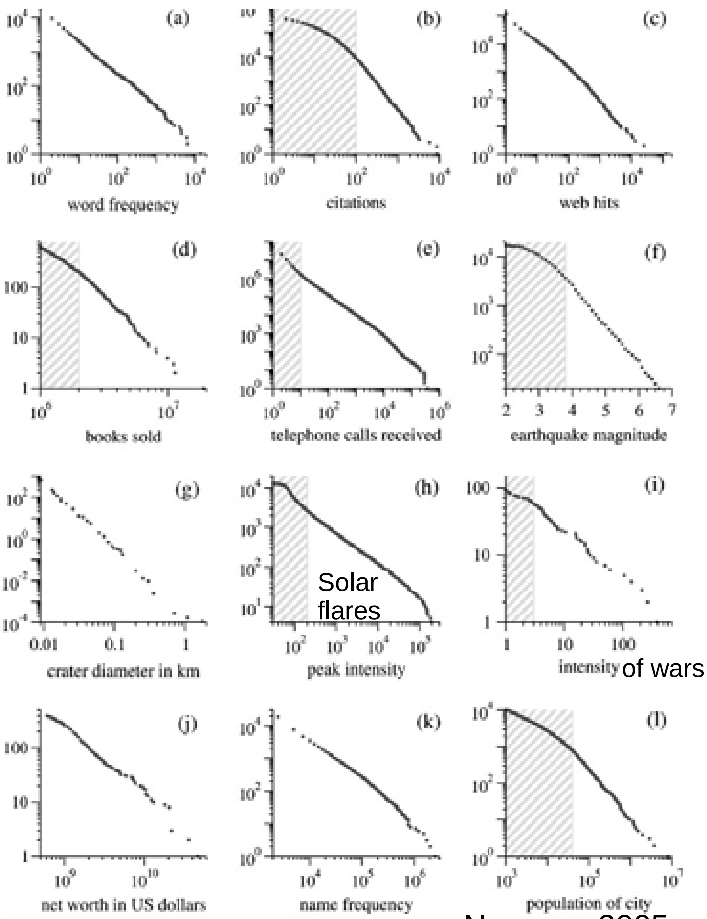
☛ Lois classiques de la thermodynamique pas valables

Ex: ~~loi 0~~: T dépend du "thermomètre" (échelle spatiale, temporelle, degrés de liberté, temps de relaxation différents..)

☛ Extension si système formé de sous-systèmes chacun en équilibre - pas vrai ici

c.f. Casa-Vasquez & Jou, 2003

Non-Maxwellian distributions: Power-law distributions



Newman 2005

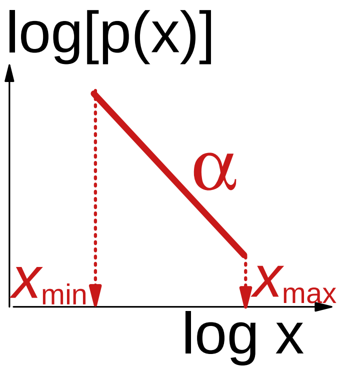
probability $p(x) \propto 1/x^{\alpha}$

	α
energy of cosmic rays	2.5
words in Moby Dick	2.2
citations to papers,	3.0
hits on web sites,	2.4
copies of books sold	3.5
telephone calls	2.2
magnitude of earthquakes	3.0
diameter of moon craters,	3.1
intensity of wars	1.8
net worth of Americans	2.1
frequency of family names,	1.9
number of species in flowering plants	2.3
etc...	

Zipf's law, Pareto distribution, ...

Power-law distributions

probability $p(x) \propto 1/x^{\alpha}$



➡ if $\alpha < 2$ $\langle x \rangle$ déterminé par x_{max}

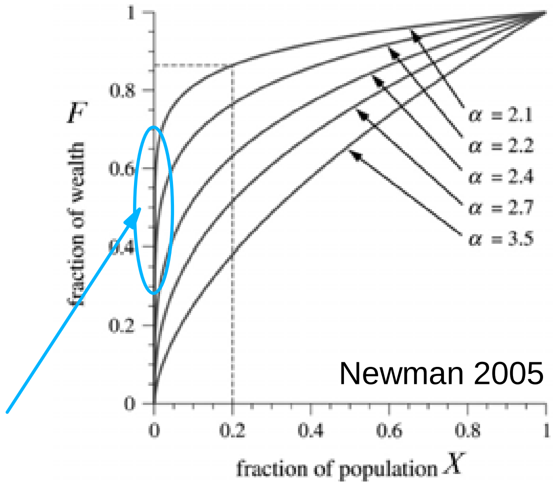
➡ if $\alpha > 2$ $\langle x \rangle$ déterminé par x_{min}

➡ if $\alpha = 2$ $\langle x \rangle \propto \ln(x_{max}/x_{min})$

Fraction *F* of the total due to the top fraction *X* of the population:

$$F = X^{(\alpha-2)/(\alpha-1)}$$

$\Rightarrow$ if $\alpha \geq 2$ a small fraction at the top holds most of the total



Newman 2005

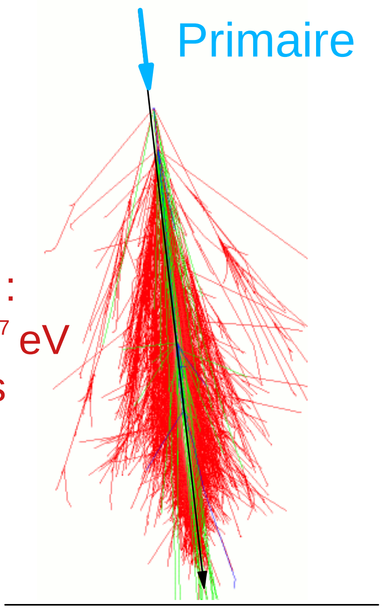
(if $\alpha \simeq 2.1$, the richest 20% hold 85% of the wealth)

Exemples en astrophysique

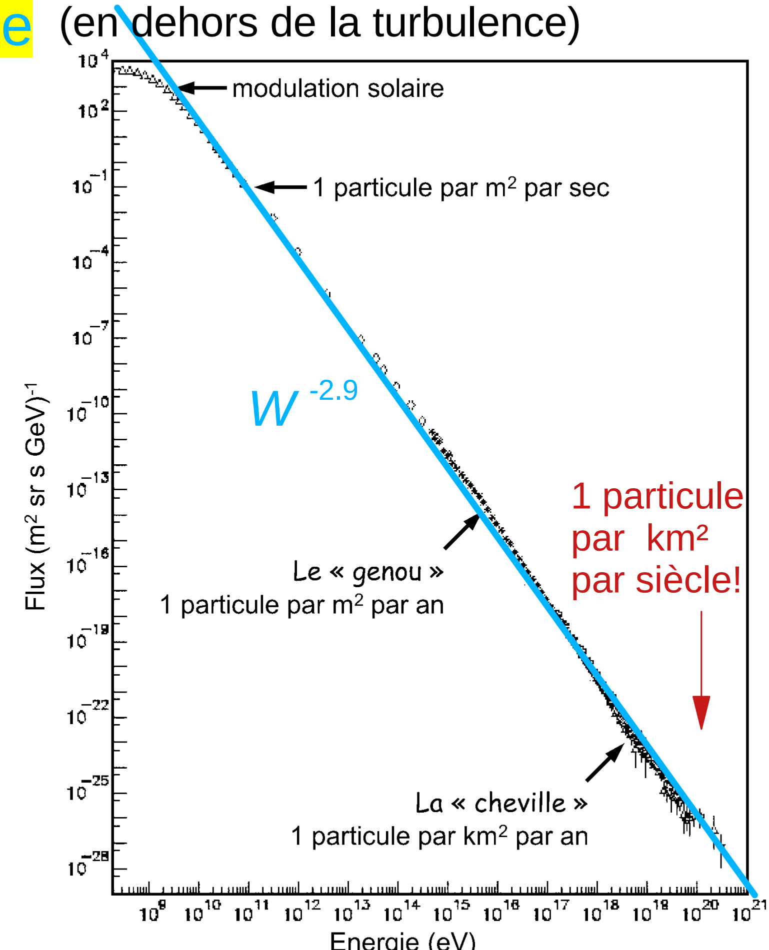
(en dehors de la turbulence)

Rayons cosmiques

gerbe atmosphérique :
1 proton de 10^{17} eV
$\rightarrow 10^{8}$ électrons

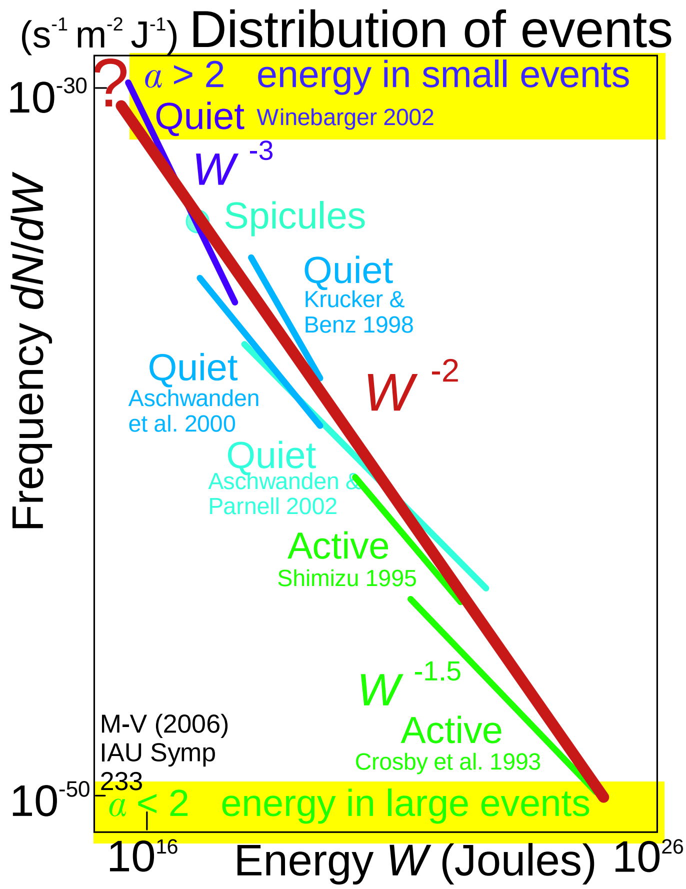


énergie des secondaires :
$dN/dW \propto 1/(W+W_0)^2$

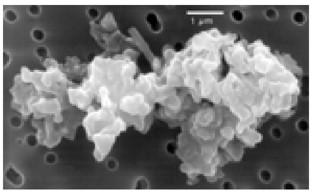


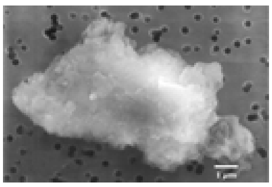
Physics Rep. 327, 109 (2000)

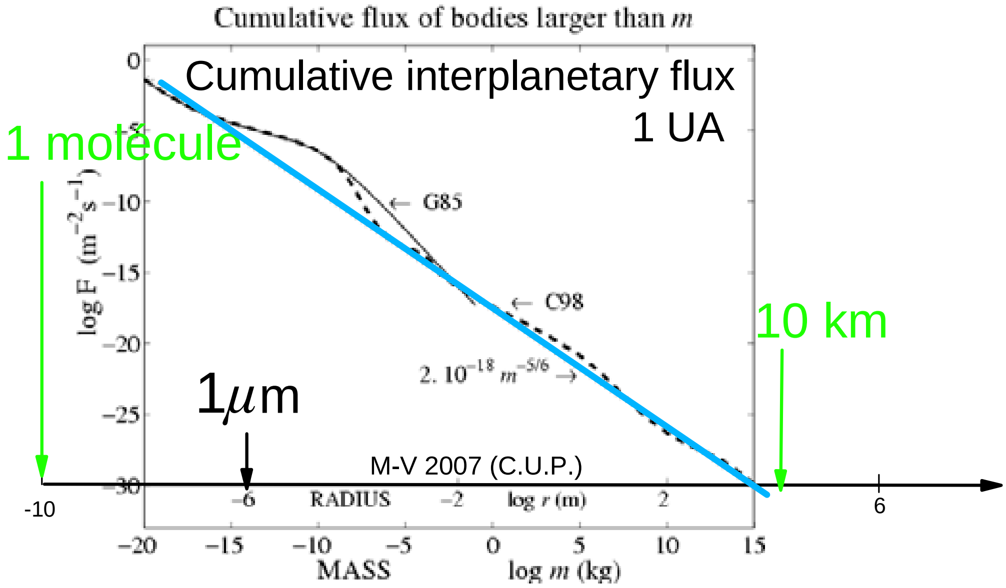
Solar activity



Distribution des objets dans le système solaire







$dN/dm \propto m^{-11/6}$

$(dN/dr \propto r^{-3.5}$ si $m \propto r^3)$

Equilibre fragmentation

Generic physics for power-law distributions ?

- Yule process ("rich" gets richer), new biological species, Fermi acceleration, etc
- Long-range correlations/Critical phenomena- forced and/or self-organized criticality Bak 1987

- ***Exponential growth:***

$$dx/dt = \alpha x \;\Rightarrow\; x = x_0\, e^{\alpha t} \;\Rightarrow\; t = \alpha^{-1} \ln(x/x_0)$$

- ***Random stop*** $P(t) = dN/dt = \nu e^{-\nu t}$ (Poisson)

$\langle t \rangle = 1/\nu$

$$\Rightarrow dN/dx \propto x^{-(1+\nu/\alpha)} \qquad x > x_0$$

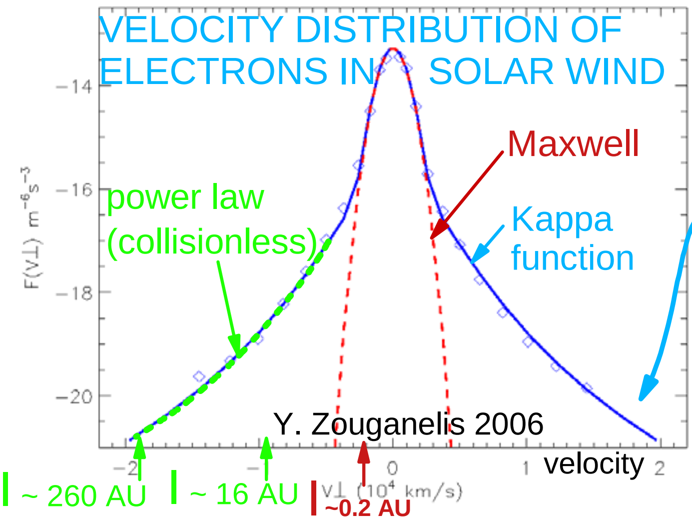
$$= \nu (x/x_0)^{-\nu/\alpha}$$

Fermi 1949, Yule 1924, Reed & Hughes 2002, ...

Particules en plasmas naturels
Fonctions de distribution observées: Kappa

$$f_\kappa(v) \propto \left(1 + \frac{v^2}{\kappa v_e^2}\right)^{-(\kappa+1)}$$

- solar wind
  - ✓ electrons: Maksimovic & al 1997, 2006
  - ✓ ions: Gloeckler & al 1992, Collier & al 1996



- magnetospheres
  - ✓ Earth: Bame et al 1967, Vasyliunas 1968, Gloeckler&Hamilton 1987, Christon&al 1989
  - ✓ Jupiter ions: Krimigis & al 1981, Hamilton & al; 1981, Kane 1991, Kane & al 1992, Collier & al 1995
    electrons: Meyer-Vernet & al 1995, Steffl & al 2004
  - ✓ Saturn: protons: Krimigis & al; 1983
  - ✓ Uranus: Krimigis & al 1986, Neptune:Mauk & al1991

- solar corona ?

- Solar wind suprathermal electrons remnants of coronal ones? Olbert 1981

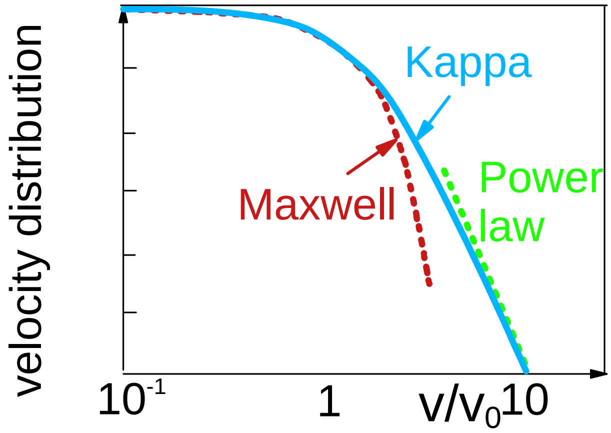
- Production of suprathermal particles (temperature grad., waves, turbulence) Roussel-Dupré 1980, Owocki & Scudder 1983, Vinas & al 2000, Vocks 2002, Vocks & Mann 2003

- Observational inferences: Dufton et al. 1984, Owocki & Ko 1999, Pinfield et al. 1999, Esser & Edgar 2000, Chiuderi & Chiuderi-Drago 2004, Doyle et al. 2004, Ko 2005 and claims to the contrary Ko et al. 1996

Generating Kappa distributions

- *Particular*

$$f_\kappa(v) \propto \left(1+\frac{v^2}{\kappa v_0^2}\right)^{-(\kappa+1)}$$



Find a way of producing a Kappa distribution in particular cases

☛ From turbulence

Roberts & Miller (1998), Vinas & al. (2000), Leubner (2000), Vocks (2002), Vocks & Mann (2003)

Difficulty : self-consistency!

Rather easy to generate *f*(*v*) from given wave spectrum ... BUT derive wave intensity from non-linear wave kinetic equation

Yoon & al. (2006) (beam-plasma with $1/nL_D^3 > 5\ 10^{-3}$)

☛ Particles ($l \gg T/\nabla T$) propagating down ∇T $\Rightarrow$ suprathermal tail (c.f. Roussel-Dupré (1980))

☛ see also Treumann 1999 (Turbulent quasi-équilibrium)

☛ see also Collier 1993 (Levy flights)

Generating Kappa distributions

- *General*

☛ *Rappel : Maxwell-Boltzmann*

"à la Jaynes" (1957) : théorie de l'information

Distribution $f(\mathbf{v})$ la plus probable sachant que

moyenne $\langle g(\mathbf{v})\rangle = U$ $\qquad \int d^3v\, f(\mathbf{v}) = 1$

On minimise $S = -k \int d^3v\, f(\mathbf{v}) \ln(f)$

"entropie" = "ignorance"
(tous les états également probables)

$\Rightarrow f(\mathbf{v}) \propto \exp[-\beta g(\mathbf{v})]$

$g(\mathbf{v}) = mv^2/2 \;\Rightarrow\; 1/\beta = 2U/3$

Cimetière de Vienne

Note: $S=k \log(W)$: c'est Planck! (Pais, 1982)

Généralisation : Hypothèses à modifier ?

- entropie additive
- énergie constante

Generating Kappa distributions

● *General*

Note: another way of generating a Maxwellian c. f. Baranger 2002

System: constant total energy U: bath + probe

Energy: $U - \varepsilon$ ε

Let: Proba. bath energy E: $\rho(E)$ with $\rho'/\rho = \beta$ for $E = U$

$\Rightarrow$ for $E \simeq U$ $\rho(E) \propto E^{\nu}$ with $\nu = \beta U$

~ effective number of degrees of freedom

$$\Rightarrow P(\varepsilon) = \rho(U - \varepsilon) \propto (U - \varepsilon)^{\nu}$$

$$\Rightarrow P(\varepsilon) \propto (1 - \varepsilon/U)^{\nu} = (1 - \beta\,\varepsilon/\nu)^{\nu} \xrightarrow[\nu \to \infty]{} e^{-\beta\varepsilon}$$ Boltzmann's factor

But if $\nu \not\gg 1$, P = Kappa function with $\kappa < 0$

finite bath

Generating Kappa distributions

● *General*

☛ *Tsallis entropy and Kappa distributions* Tsallis 1988-1998, 2004

- Boltzmann-Gibbs entropy $S = k_B < \ln(1/f) >$ additive
- Tsallis entropy $S_q = k_B < \ln_q(1/f) >$ non-additive

$$\ln_q(f) = (f^{1-q} - 1)/(1-q) \xrightarrow[q \to 1]{} \ln(f)$$

$$\langle g \rangle = \int dx\, f^q g / \int dx\, f^q$$

$$x^p = e^{p \ln(x)} \underset{p \to 0}{\sim} 1 + p \ln(x)$$

Minimize S with $<E> = U$

$\Rightarrow f(E) \propto \exp_q(-\beta E)$ $\exp_q(x) = [1+(1-q)x]^{1/(1-q)} \xrightarrow[q \to 1]{} \exp(x)$ 1-D

$\Rightarrow$ Kappa dist. with $\kappa = 1/(q-1)$

Physical meaning of q ? Non-additive entropy*, i.e. possible bias in the distribution .. BUT should be derived from first principles

(* ex.: merging two indep. turb. systems $\Rightarrow$ change R $\Rightarrow$ change turbulent behavic

Presently not a genuine theory but "a description which works"

c.f. Balmer formula for spectral lines, Ptolemy and Copernicus epicycles ...

☛ *Super-statistics (variation in "temperature")*

- ➤ System not isolated
- ➤ Non-infinite heat bath

- **Temperature fluctuations** $\Rightarrow$ Tsallis (Kappa) distribution if β has gamma (chi2) distribution

$$[1+(q-1)\beta_0 E]^{-1/(q-1)} = \int d\beta\, e^{-\beta E} f(\beta) \qquad <\beta> = \beta_0 \qquad q = <\beta^2>/<\beta>^2$$

statistics ($f(\beta)$) of a statistics ($e^{-\beta E}$)

Wilk & Wlodarczyk 2000, Beck 2002, Cohen 2004

$\Rightarrow$ *q* not arbitrary: determined by temperature fluctuations

- "Small heat bath" statistics (Almeida 2001)

➤ implicit in Boltzmann ($q = 1$): temperature of heat bath independent on its energy *U* (infinite heat capacity)

➤ if temperature depends on *U*, then: $q - 1 = d(1/\beta)/dU$ inverse of heat capacity

$\Rightarrow$ *q* not arbitrary: determined by variation of temperature with energy (q>1 : positive heat capacity; q<1 : negative heat capacity)

☛ Collisionless particles in a potentiel Φ

Particles in an attractive force (gravity + electric field)

Liouville theorem

$\Rightarrow f(W) = f_0(W + \Delta\psi)$ potential energy

$\Rightarrow$ With Maxwellian: *T* constant

Kappa $\quad f_0(v) \propto [1+v^2/\kappa v_0^2]^{-\kappa}$

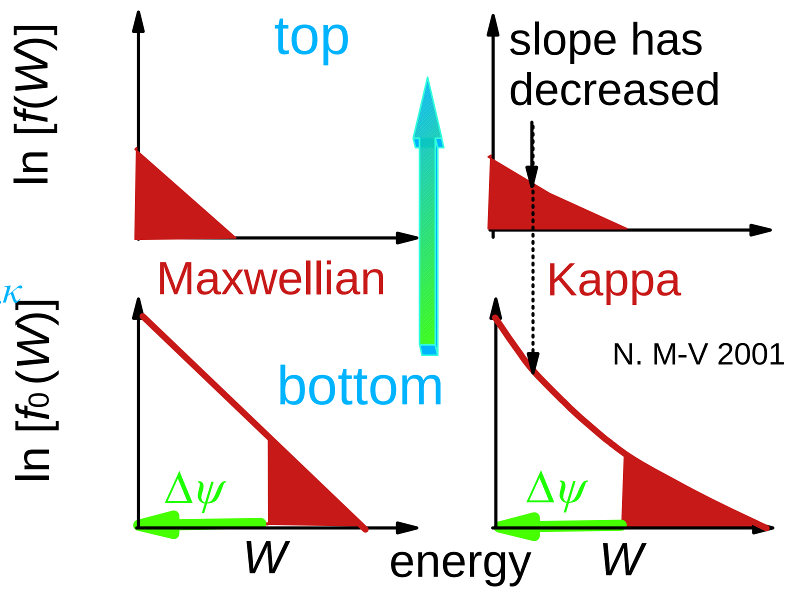
$$\Rightarrow f(v,z) \propto [1+(v^2+2\Delta\psi/m)/\kappa v_0^2]^{-\kappa}$$

$$\Rightarrow f(v,z) = t^{-\kappa} f_0(v t^{-1/2})$$

$$t = 1+2\Delta\psi/m\kappa v_0^2$$

$$T \propto t$$

$\Rightarrow$ With Kappa distribution: *T* increases with potential energy Scudder 1992



No heating: velocity filtration instead (attractive force lets suprathermal particles escape)

Collisionless particles in a potential Φ

Kappa $f_0(v) \propto [1+v^2/\kappa v_0^2]^{-\kappa}$ $\Rightarrow f(v,z) \propto [1+(v^2+2\Phi/m)/\kappa v_0^2]^{-\kappa}$

$\Rightarrow f(v,z) = t^{-\kappa} f_0(vt^{-1/2})$ $t = 1+\Phi/[\kappa(mv_0^2/2)]$

Temperature: $T \propto t$ Density: $n \propto T^{1/2-\kappa}$

Define: $k_B T_0 = mv_0^2/2$

$\Rightarrow k_B dT/d\Phi = 1/\kappa$ Maxwellian : $\kappa \rightarrow \infty$ $dT/d\Phi \rightarrow 0$

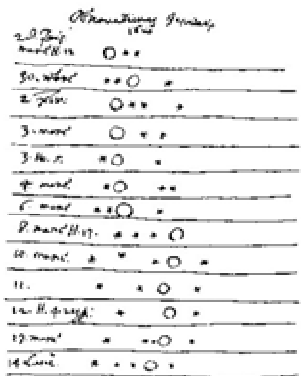
c.f. "small heat bath" statistics : $q - 1 = d(1/\beta)/dU$

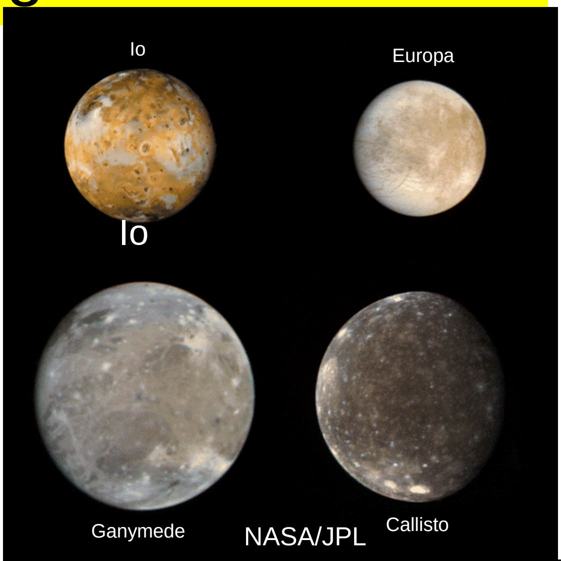
$\beta = 1/k_B T$ $\kappa = 1/(q-1)$ systems with long-range interactions

polytrope law $p \propto n^{\gamma}$ $\gamma = 1 - 1/(\kappa - 1/2) < 1$

Application sur une structure grande échelle

Jupiter



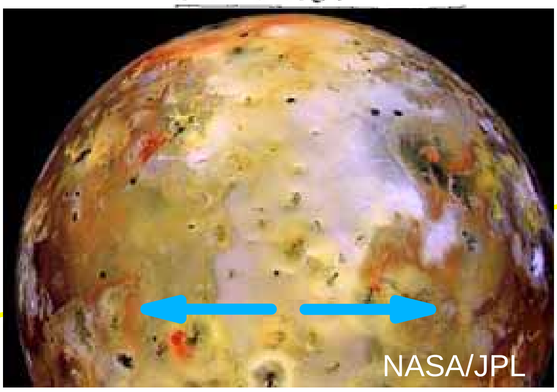


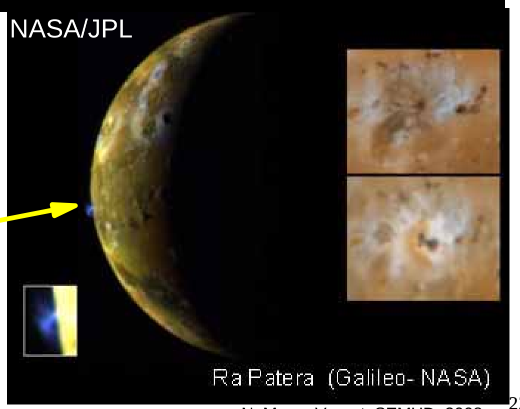
Io: marées gigantesques

- rotation = révolution
- orbite eccentrique
- résonance orbites Europa, Ganymède

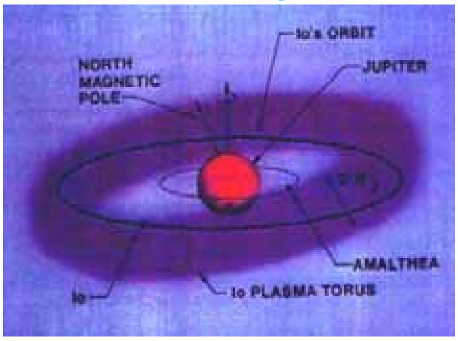
R ~ 1800 km, r from Jupiter ~ 4 10^5 km

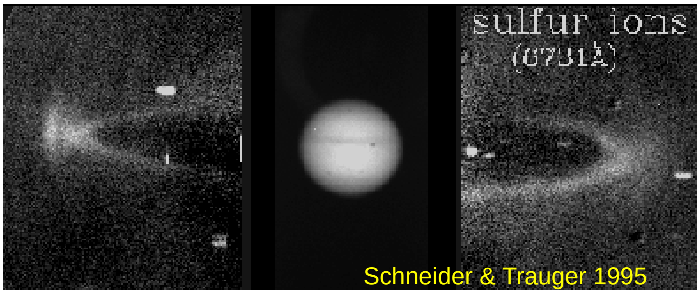
$\Rightarrow$ Volcanisme



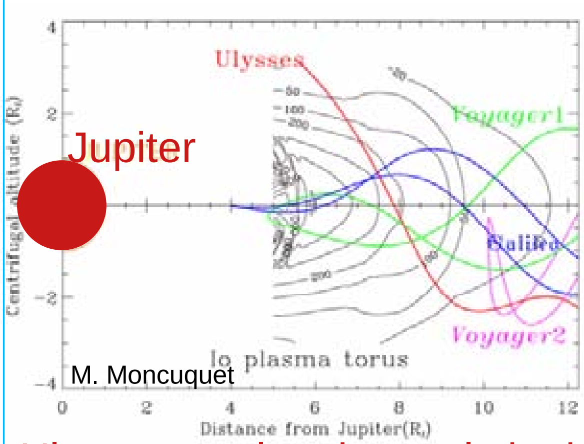


Particules ↦ ionisation ↦ pick-up par champ magnétique ↦ Tore de plasma

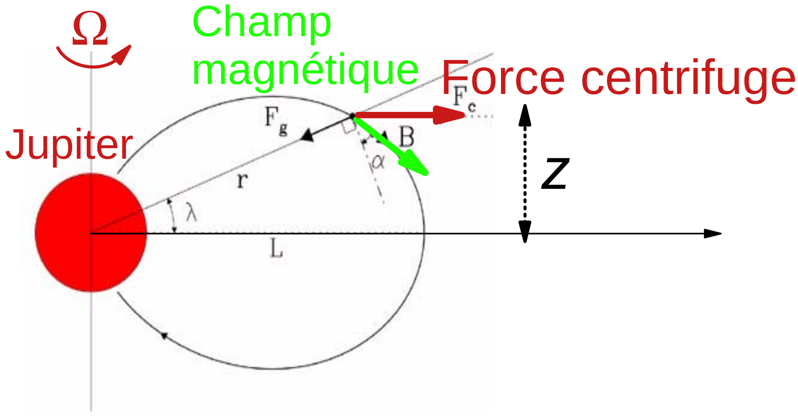




Visiteurs du tore



Ulysses: exploration en latitude



Potentiel : $\Phi_c = 3m\Omega^2 z^2/2$

Champ électrique de polarisation ⇒ potentiel total Φ_{tot}

Maxwellienne

$f_0(v) \propto \exp(-mv^2/2k_BT)$ ➡ $f(v,z) = f_0(v)\ \exp[-\Phi_{tot}(z)/k_BT]$

Kappa

$$f_0(v) \propto \left[1 + \frac{mv^2/2}{(\kappa-3/2)k_BT_0}\right]^{-(\kappa+1)}$$

$$f(v,z) \propto \left[1 + \frac{mv^2/2 + \Phi_{tot}(z)}{(\kappa-3/2)k_BT_0}\right]^{-(\kappa+1)}$$

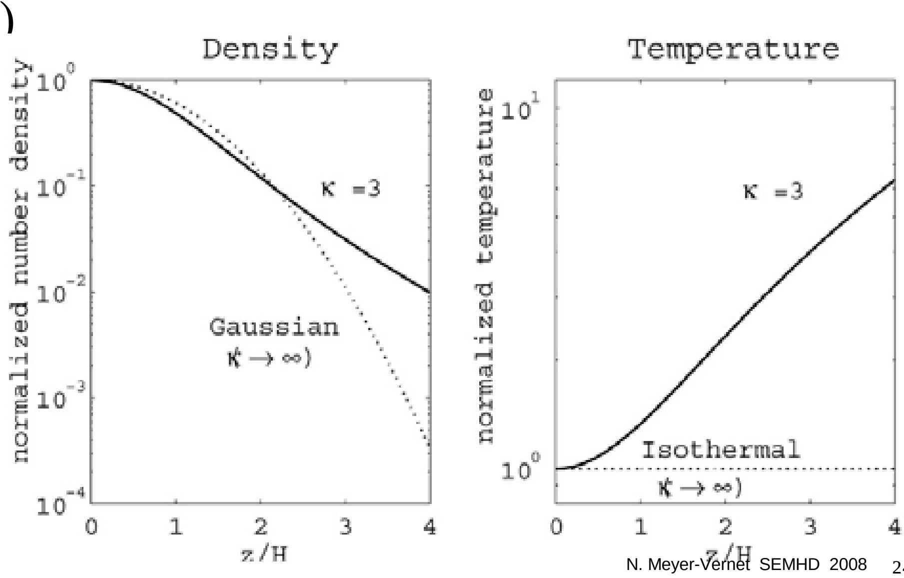
➡ $f(v,z) = t^{-(\kappa+1)} f_0(v t^{-1/2})$

$$t = 1 + \frac{\Phi_{tot}(z)}{(\kappa-3/2)k_BT_0}$$

➡ $T \propto t \propto n^{-1/(\kappa-1/2)}$

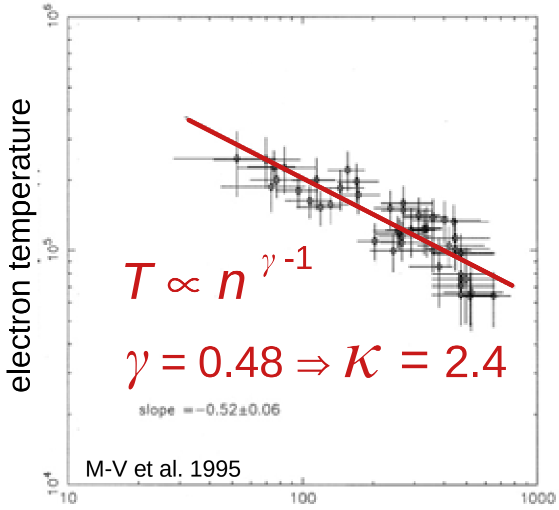
Note: anisotropic Kappa : $T_{\parallel}$ increases with altitude (Moncuquet et al.,2002)

Sum of several maxwellians : T increases with altitude (M-V et al., 1995)



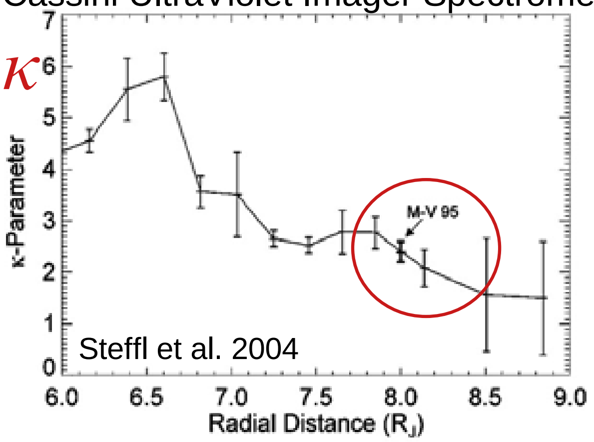
$\Rightarrow$ polytrope law $p \propto n^{\gamma}$

$\gamma = 1 - 1/(\kappa - 1/2)$



Confirmation par mesures spectres UV

Cassini UltraViolet Imager Spectrometer



Hubble Space Telescope Imaging Spectrograph