

Structures hors équilibre à grande échelle en plasmas naturels : limites de la MHD (2)

or: "The importance of not being Maxwellian"

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SEMHD

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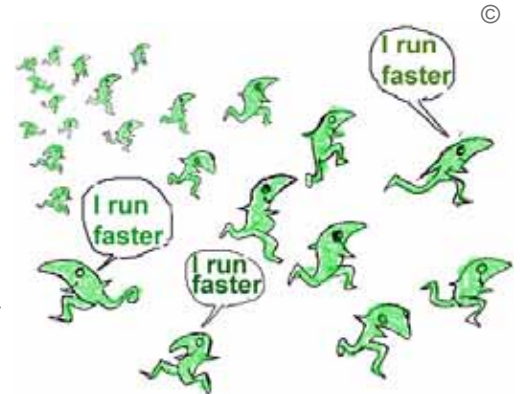
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1

What fluid (MHD) models are doing:

Basically: replace eqs. of motion of $N \gg 1$ particles

by **a few local** differential eqs. on **a few** macroscopic parameters (n, V, T, \dots)



Planet. Space Sci. 49 247 (2001)

- continuity equation
- momentum equation
- energy equation
-

Infinite hierarchy of coupled differential equations

👉 **Closing the hierarchy:**

- assume nearly (bi-)Maxwellian ($I \ll H$)
- OR ad-hoc **heat transfer**, or ..



Each species: one fluid with one T (possibly $T_{\parallel} \neq T_{\perp}$)



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Niveaux de description

Cas classique :

$$v \ll c$$

$$d \sim n^{-1/3} \gg h/mv$$

● N particules individuelles

- **état** : r, v pour chaque particule à t
- **évolution** : éq. mouv. de chaque particule
- *mécanique analytique/constants du mouv.*

● fonction de distribution

- **état** : $f(r, v)$ à t (proba. de trouver particules avec position r , vitesse v)
- **évolution** : éq. d'évolution de $f(r, v)$
- *mécanique statistique*

● fluide

- **état** : **quelques** grandeurs macroscopiques n, V, P, \dots de l'élément de fluide en r, t
- **évolution** : éqs. de conservation (nombre de particules, qté de mouv., énergie, ..)
- \Rightarrow *équations différentielles locales*

description impossible si N trop grand (quoique ...)

en général f défini par une infinité de paramètres

défini par petit nombre de paramètres

moyennes



moyennes

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3

Problème: Fermeture de la hiérarchie: Equation d'énergie

Cas le plus simple: Pression isotrope $P, \frac{\partial}{\partial t} = 0$

● $P \propto \rho^\gamma \Rightarrow$ (Bernouilli) $\mathbf{V} \cdot \nabla \left(\frac{V^2}{2} + \frac{\gamma}{\gamma-1} \frac{k_B T}{m} + \Phi_G \right) = 0$

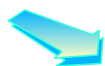
ou:

● Flux de chaleur $Q \neq 0$: $\rho \mathbf{V} \cdot \nabla \left(\frac{V^2}{2} + \frac{\gamma}{\gamma-1} \frac{k_B T}{m} + \Phi_G \right) = -\nabla \cdot \mathbf{Q}$

$\mathbf{Q} = -\mathbf{K} \nabla T$ si $l_{pm} \ll L \sim T / \nabla T$

↑ conductivité thermique classique

$\mathbf{K} \sim (3/2) n k_B v_{the} l_{pm}$



$\frac{\text{conduction}}{\text{advection}} \sim \frac{\kappa T / L^2}{n V k_B T / L} \sim \frac{v_{the} l_{pm}}{V L}$

$\Rightarrow \nabla \cdot \mathbf{Q}$ domine

MAIS : $\mathbf{K} \neq$ valeur classique dès que $l_{pm} / L > 10^{-3}$

(Scudder & Olbert 1983, Shoub 1983, Canullo & al. 1996, Dorelli & Scudder, 1999, 2003, Pantellini & Landi 2001)

- Car :
- Q déterminé par électrons suprathermiques
 - électrons suprathermiques non collisionnels ($l \gg l_{pm}$)

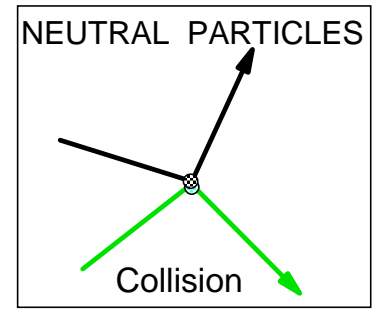
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4

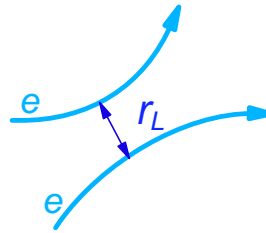
Collisions : Difference between plasmas and neutral gases

Neutral particles

collisional cross-section "physical section"
in order of magnitude



Charged particles : Coulomb interactions



For a (relative) speed v , a significant perturbation requires distance r_L such that:

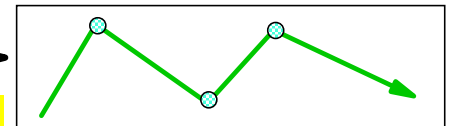
$$\frac{e^2}{4\pi\epsilon_0 r_L} \sim m v^2$$

Coulomb energy

cross section : $\sigma_C \sim r_L^2 \Rightarrow$ free path $l(v) \propto v^4$
 $(\times \ln \Lambda) \propto 1/v^4 \Rightarrow$ if $v \times 3$ then $l(v) \times 100!$

fast particles are collisionless, even when most particles (the core of the velocity distribution) are collisional

Trajectoire d'une particule neutre



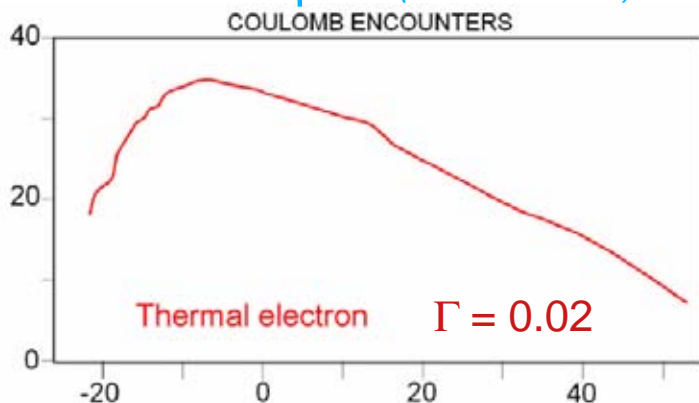
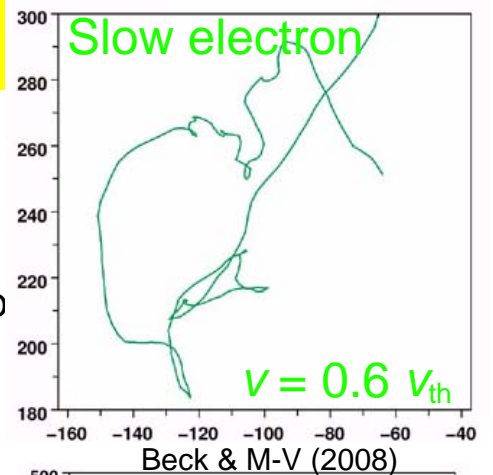
Trajectoire d'un électron en plasma gazeux

$r_L \ll \langle r \rangle \sim n^{-1/3} \Rightarrow$ la plupart des interactions produisent petites perturbations

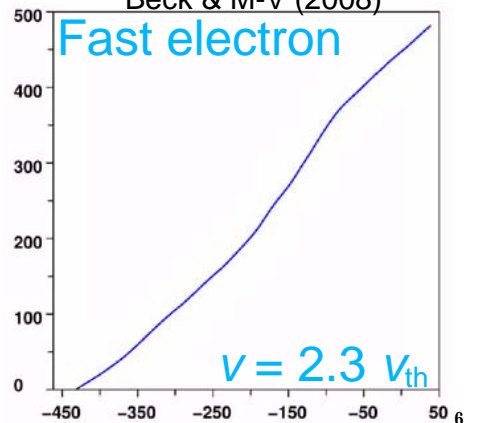
$$\Gamma \equiv \frac{e^2}{4\pi\epsilon_0 \langle r \rangle k_B T} \ll 1$$

$L_D / \langle r \rangle = (4\pi\Gamma)^{-1/2} \gg 1 \Rightarrow$ 1 particule subit beaucoup d'interactions simultanément

simulation n-corps (Arnaud Beck)



unité: distance moyenne entre particules



- fast particles are collisionless
- various acceleration processes



fast particles are not Maxwellian
 ⇒ fonction de distribution ≠ Maxwellian



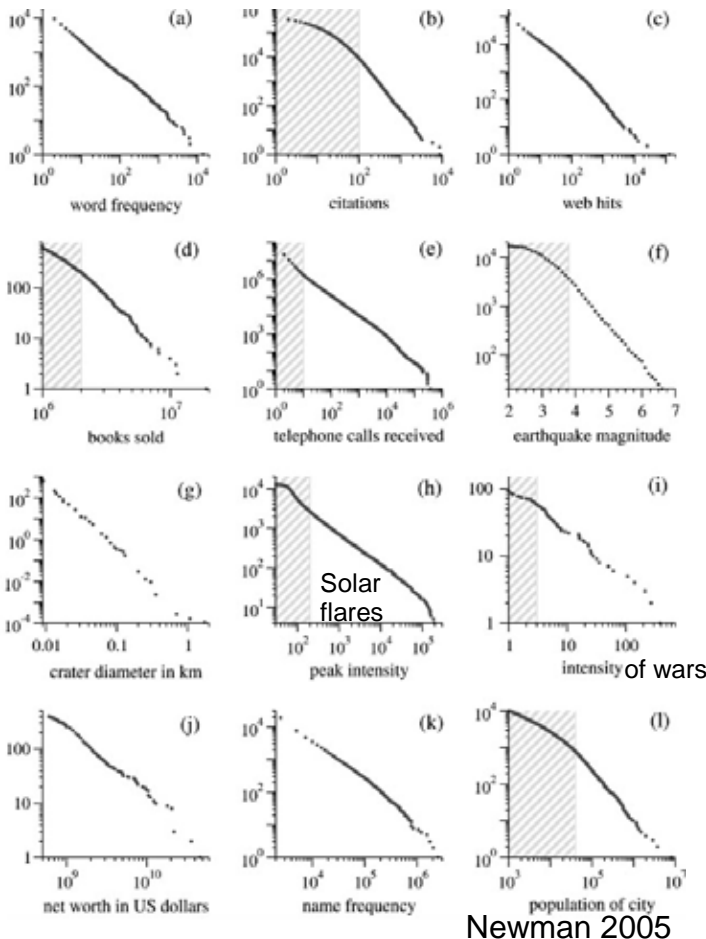
Température hors équilibre local ?

Par ex. : Température cinétique : $\frac{3}{2}k_B T = \int d^3 v (mv^2/2) f(\mathbf{v})$
 dans repère $\langle \mathbf{v} \rangle = 0$ normalisation $\int d^3 v f(\mathbf{v}) = 1$

- Lois classiques de la thermodynamique pas valables
 Ex: loi 0: ~~T~~ dépend du "thermomètre" (échelle spatiale, temporelle, degrés de liberté, temps de relaxation différents..)
- Extension si système formé de sous-systèmes chacun en équilibre - pas vrai ici

c.f. Casa-Vasquez & Jou, 2003

Non-Maxwellian distributions: Power-law distributions



probability $p(x) \propto 1/x^\alpha$

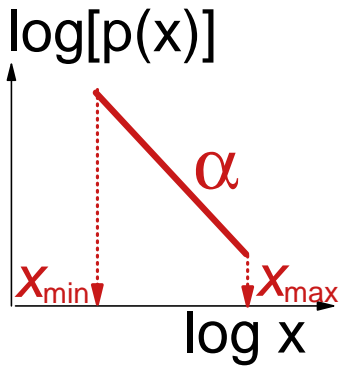
- | | α |
|---|----------|
| ● energy of cosmic rays | 2.5 |
| ● words in Moby Dick | 2.2 |
| ● citations to papers, | 3.0 |
| ● hits on web sites, | 2.4 |
| ● copies of books sold | 3.5 |
| ● telephone calls | 2.2 |
| ● magnitude of earthquakes | 3.0 |
| ● diameter of moon craters, | 3.1 |
| ● intensity of wars | 1.8 |
| ● net worth of Americans | 2.1 |
| ● frequency of family names, | 1.9 |
| ● number of species in flowering plants | 2.3 |
| ● etc... | |

Zipf's law, Pareto distribution, ...

Newman 2005

Power-law distributions

probability $p(x) \propto 1/x^\alpha$



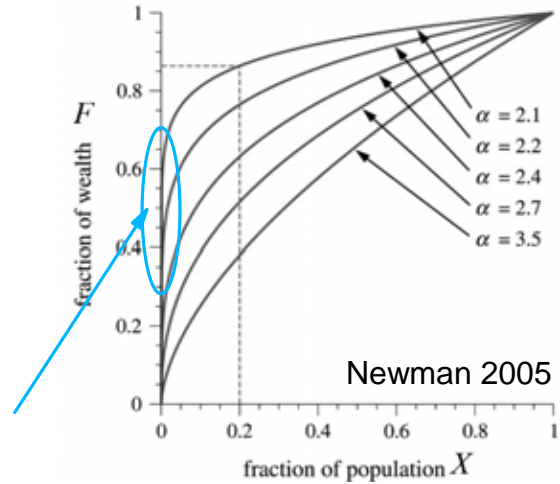
- if $\alpha < 2$ $\langle x \rangle$ déterminé par x_{\max}
- if $\alpha > 2$ $\langle x \rangle$ déterminé par x_{\min}
- if $\alpha = 2$ $\langle x \rangle \propto \ln(x_{\max}/x_{\min})$

Fraction F of the total due to the top fraction X of the population:

$$F = X^{(\alpha - 2)/(\alpha - 1)}$$

⇒ if $\alpha \geq 2$ a small fraction at the top holds most of the total

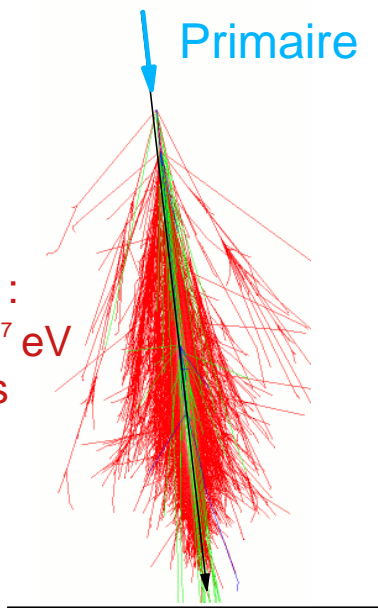
(if $\alpha \approx 2.1$, the richest 20% hold 85% of the wealth)



Exemples en astrophysique

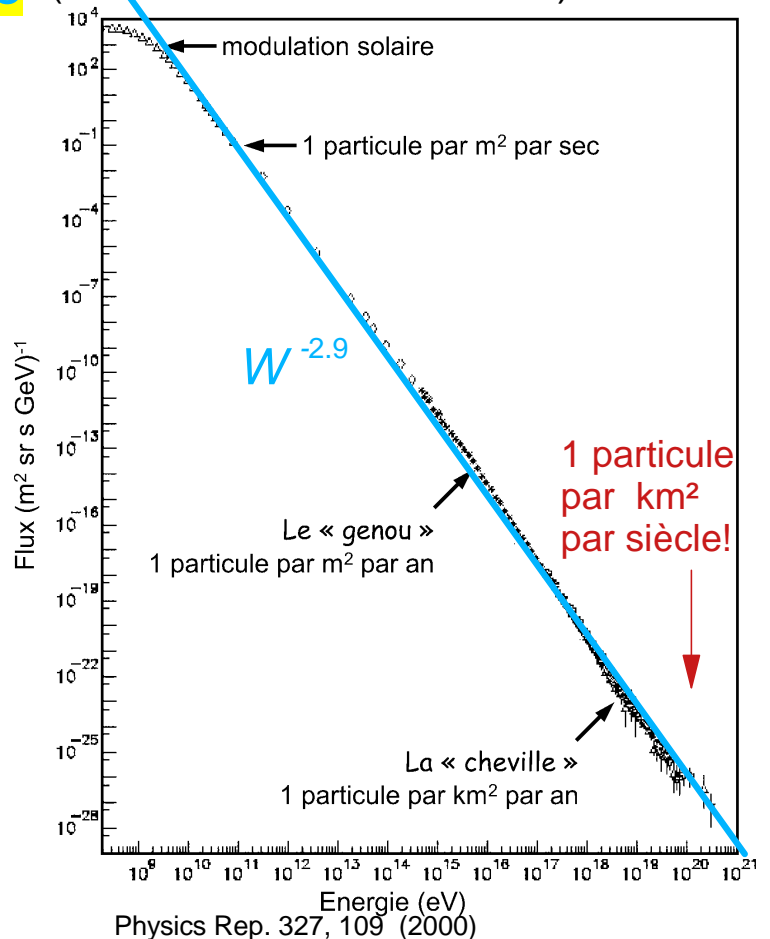
Rayons cosmiques

gerbe atmosphérique :
1 proton de 10^{17} eV
→ 10^8 électrons

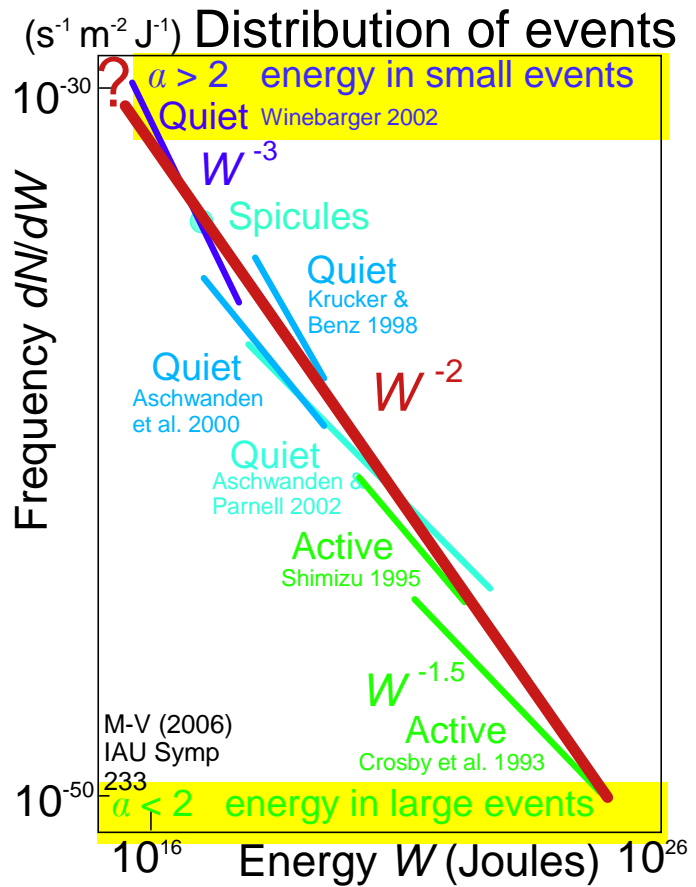
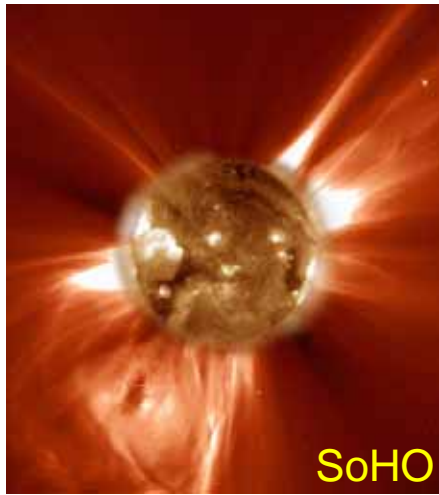


énergie des secondaires :
 $dN/dW \propto 1/(W+W_0)^2$

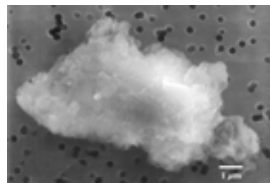
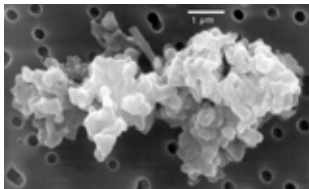
(en dehors de la turbulence)



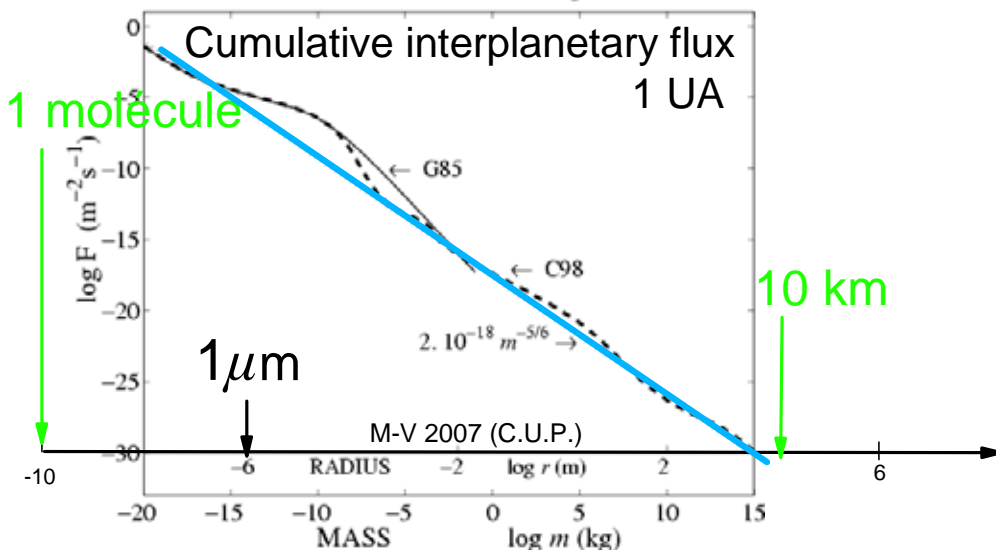
Solar activity



Distribution des objets dans le système solaire



Cumulative flux of bodies larger than m



Itokawa Hayabusa (Jare)

$$dN/dm \propto m^{-11/6}$$

$$(dN/dr \propto r^{3.5} \text{ si } m \propto r^3)$$

Equilibre fragmentation

Generic physics for power-law distributions ?

- Yule process ("rich" gets richer), new biological species, Fermi acceleration, etc
- Long-range correlations/Critical phenomena- forced and/or self-organized criticality Bak 1987

Exponential growth:

$$dx/dt = \alpha x \Rightarrow x = x_0 e^{\alpha t} \Rightarrow t = \alpha^{-1} \ln(x/x_0)$$

Random stop $P(t) = dN/dt = \nu e^{-\nu t}$ (Poisson)

$$\begin{aligned} \Rightarrow dN/dx &\propto x^{-(1+\nu/\alpha)} & x > x_0 \\ &= \nu (x/x_0)^{-\nu/\alpha} \end{aligned} \quad \langle t \rangle = 1/\nu$$

Fermi 1949, Yule 1924, Reed & Hughes 2002, ...
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Particules en plasmas naturels

Fonctions de distribution observées: Kappa

$$f_{\kappa}(v) \propto \left(1 + \frac{v^2}{\kappa v_e^2}\right)^{-(\kappa+1)}$$

● solar wind

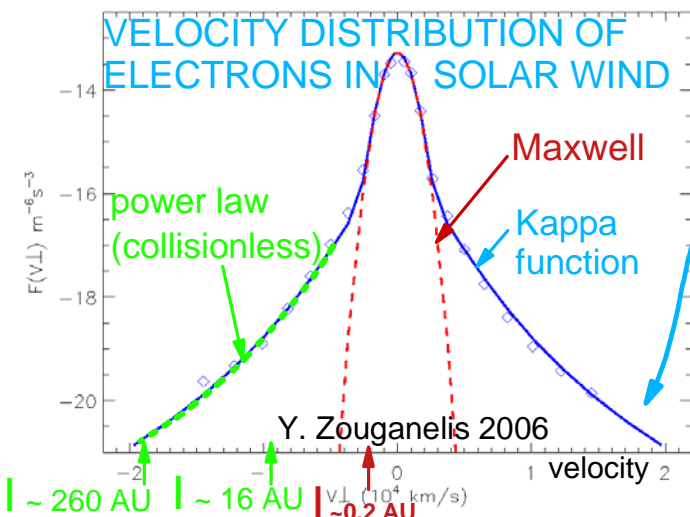
- ✓ electrons: Maksimovic & al 1997, 2006
- ✓ ions: Gloeckler & al 1992, Collier & al 1996

● magnetospheres

- ✓ Earth: Bame et al 1967, Vasyliunas 1968, Gloeckler&Hamilton 1987, Christon&al 1989
- ✓ Jupiter ions: Krimigis & al 1981, Hamilton & al; 1981, Kane 1991, Kane & al 1992, Collier & al 1995
electrons: Meyer-Vernet & al 1995, Steffl & al 2004
- ✓ Saturn: protons: Krimigis & al; 1983
- ✓ Uranus: Krimigis & al 1986, Neptune: Mauk & al 1991

● solar corona ?

- Solar wind suprathermal electrons remnants of coronal ones? Olbert 1981
- Production of suprathermal particles (temperature grad., waves, turbulence) Roussel-Dupré 1980, Owocki & Scudder 1983, Vinas & al 2000, Vocks 2002, Vocks & Mann 2003
- Observational inferences: Dufton et al. 1984, Owocki & Ko 1999, Pinfield et al. 1999, Esser & Edgar 2000, Chiuderi & Chiuderi-Drago 2004, Doyle et al. 2004, Ko 2005 and claims to the contrary Ko et al. 1996



Generating Kappa distributions

● Particular

Find a way of producing a Kappa distribution in particular cases

➤ From turbulence

Roberts & Miller (1998), Vinas & al. (2000), Leubner (2000), Vocks (2002), Vocks & Mann (2003)

Difficulty : self-consistency!

Rather easy to generate $f(v)$ from given wave spectrum ... BUT derive wave intensity from non-linear wave kinetic equation

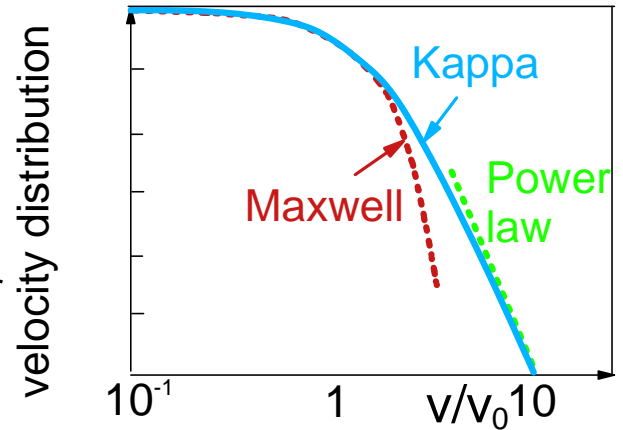
Yoon & al. (2006) (beam-plasma with $1/nL_D^3 > 5 \cdot 10^{-3}$)

➤ Particles ($I \gg T/\nabla T$) propagating down $\nabla T \Rightarrow$ suprathermal tail (c.f. Roussel-Dupré (1980))

➤ see also Treumann 1999 (Turbulent quasi-équilibre)

➤ see also Collier 1993 (Levy flights)

$$f_\kappa(v) \propto \left(1 + \frac{v^2}{\kappa v_\theta^2}\right)^{-(\kappa+1)}$$



Generating Kappa distributions

➤ Rappel : Maxwell-Boltzmann

"à la Jaynes" (1957) : théorie de l'information

Distribution $f(\mathbf{v})$ la plus probable sachant que

$$\text{moyenne } \langle g(\mathbf{v}) \rangle = U \quad \int d^3v f(\mathbf{v}) = 1$$

On minimise $S = -k \int d^3v f(\mathbf{v}) \ln(f)$

"entropie" = "ignorance"

(tous les états également probables)

$$\Rightarrow f(\mathbf{v}) \propto \exp[-\beta g(\mathbf{v})]$$

$$g(\mathbf{v}) = mv^2/2 \Rightarrow 1/\beta = 2U/3$$

Généralisation : Hypothèses à modifier ?

● General



Cimetière de Vienne

Note: $S = k \log(W)$: c'est Planck! (Pais, 1982)

- entropie additive
- énergie constante

Generating Kappa distributions

● General

Note: another way of generating a Maxwellian

c. f. Baranger 2002

System: constant total energy U : bath + probe

Energy: $U - \varepsilon$ ε

Let: Proba. bath energy E : $\rho(E)$ with $\rho'/\rho = \beta$ for $E = U$

\Rightarrow for $E \approx U$ $\rho(E) \propto E^\nu$ with $\nu = \beta U$
 \nearrow ~ effective number of degrees of freedom

$$\Rightarrow P(\varepsilon) = \rho(U - \varepsilon) \propto (U - \varepsilon)^\nu$$

$$\Rightarrow P(\varepsilon) \propto (1 - \varepsilon/U)^\nu = (1 - \beta \varepsilon/\nu)^\nu \xrightarrow{\nu \rightarrow \infty} e^{-\beta \varepsilon} \quad \text{Boltzmann's factor}$$

But if $\nu \not\gg 1$, $P =$ Kappa function with $\kappa < 0$

finite bath

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Generating Kappa distributions

● General

➤ Tsallis entropy and Kappa distributions Tsallis 1988-1998, 2004

- Boltzmann-Gibbs entropy $S = k_B \langle \ln(1/f) \rangle$ additive

- Tsallis entropy $S_q = k_B \langle \ln_q(1/f) \rangle$ non-additive

$$\ln_q(f) = (f^{1-q} - 1)/(1-q) \quad \langle g \rangle = \int dx f^q g / \int dx f^q$$

$$\xrightarrow{q \rightarrow 1} \ln(f) \quad x^p = e^{p \ln(x)} \sim 1 + p \ln(x) \quad p \rightarrow 0$$

Minimize S with $\langle E \rangle = U$

$$\Rightarrow f(E) \propto \exp_q(-\beta E) \quad \exp_q(x) = [1 + (1-q)x]^{1/(1-q)} \xrightarrow{q \rightarrow 1} \exp(x) \quad \text{1-D}$$

\Rightarrow Kappa dist. with $\kappa = 1/(q-1)$

Physical meaning of q ? Non-additive entropy*, i.e. possible bias in the distribution .. BUT should be derived from first principles

(* ex.: merging two indep. turb. systems \Rightarrow change $R \Rightarrow$ change turbulent behavior)

Presently not a genuine theory but "a description which works"

c.f. Balmer formula for spectral lines, Ptolemy and Copernicus epicycles ...

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➤ Super-statistics (variation in "temperature")

- ▶ System not isolated
- ▶ Non-infinite heat bath

- **Temperature fluctuations** \Rightarrow Tsallis (Kappa) distribution if β has gamma (chi2) distribution

$$[1+(q-1)\beta_0 E]^{-1/(q-1)} = \int d\beta e^{-\beta E} f(\beta) \quad \langle \beta \rangle = \beta_0 \quad q = \langle \beta^2 \rangle / \langle \beta \rangle^2$$

statistics ($f(\beta)$) of a statistics ($e^{-\beta E}$)

Wilk & Włodarczyk 2000,
Beck 2002, Cohen 2004

\Rightarrow q not arbitrary: determined by temperature fluctuations

- "Small heat bath" statistics (Almeida 2001)

- ▶ implicit in Boltzmann ($q=1$): temperature of heat bath independent on its energy U (infinite heat capacity)
- ▶ if temperature depends on U , then: $q-1 = d(1/\beta)/dU$ inverse of heat capacity
- \Rightarrow q not arbitrary: determined by variation of temperature with energy ($q>1$: positive heat capacity; $q<1$: negative heat capacity)

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➤ Collisionless particles in a potential Φ

Particles in an attractive force (gravity + electric field)

Liouville theorem

$$\Rightarrow f(W) = f_0(W + \Delta\psi) \quad \text{potential energy}$$

\Rightarrow With Maxwellian: T constant

Kappa $f_0(v) \propto [1+v^2/\kappa v_0^2]^{-\kappa}$

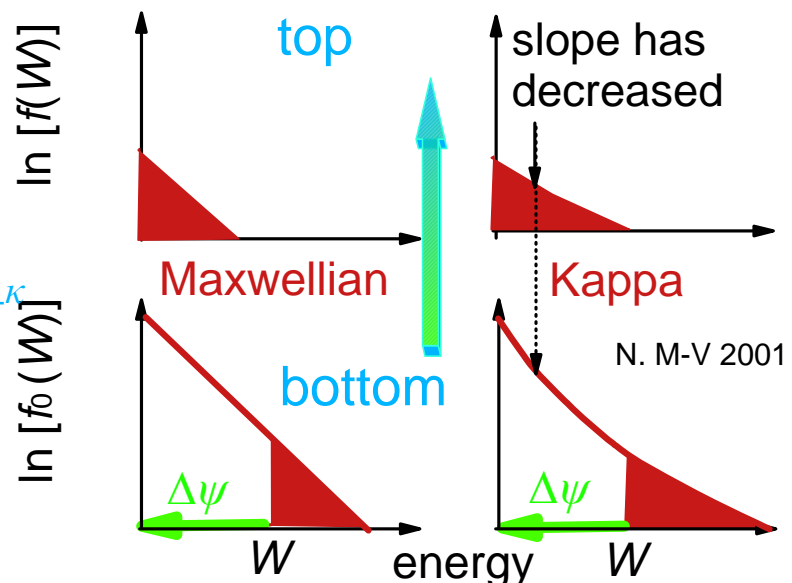
$$\Rightarrow f(v,z) \propto [1+(v^2+2\Delta\psi/m)/\kappa v_0^2]^{-\kappa}$$

$$\Rightarrow f(v,z) = t^{-\kappa} f_0(vt^{1/2})$$

$$t = 1 + 2\Delta\psi/m\kappa v_0^2$$

$$T \propto t$$

\Rightarrow With Kappa distribution:
 T increases with potential energy
Scudder 1992



No heating: velocity filtration instead
(attractive force lets suprathermal particles escape)

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Collisionless particles in a potential Φ

$f_0(v) \propto [1+v^2/\kappa v_0^2]^{-\kappa} \Rightarrow f(v,z) \propto [1+(v^2+2\Phi/m)/\kappa v_0^2]^{-\kappa}$
 $\Rightarrow f(v,z) = t^{-\kappa} f_0(vt^{1/2}) \quad t = 1+\Phi/[\kappa(mv_0^2/2)]$
 Temperature: $T \propto t$ Density: $n \propto T^{1/2-\kappa}$

Define: $k_B T_0 = mv_0^2/2$

$\Rightarrow k_B dT/d\Phi = 1/\kappa$

Maxwellian : $\kappa \rightarrow \infty \quad dT/d\Phi \rightarrow 0$

c.f. "small heat bath" statistics : $q-1 = d(1/\beta)/dU$

$\beta = 1/k_B T \quad \kappa = 1/(q-1)$ systems with long-range interactions

polytrope law $p \propto n^\gamma \quad \gamma = 1 - 1/(\kappa - 1/2) < 1$

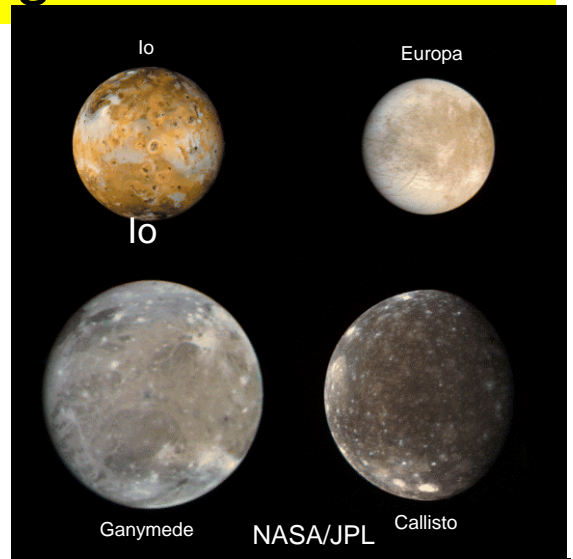
Application sur une structure grande échelle

Jupiter



Observations Jovian
20 July 1872

Io	○ ○ ○
Europa	○ ○ ○
Ganymede	○ ○ ○
Callisto	○ ○ ○
Jupiter	○ ○ ○
Other	○ ○ ○

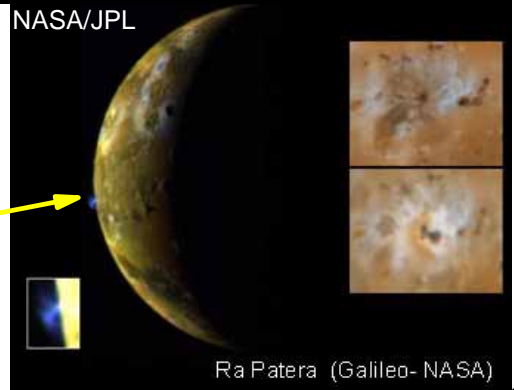
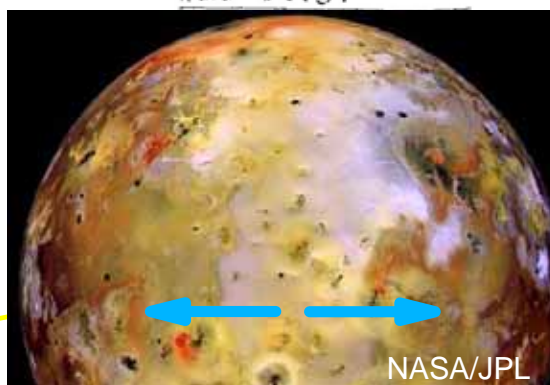


Io: marées gigantesques

- rotation = révolution
- orbite eccentric
- résonance orbites Europa, Ganymède

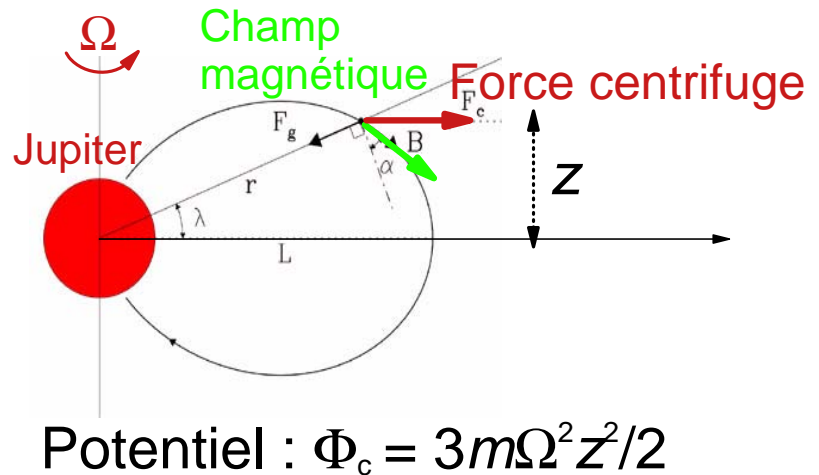
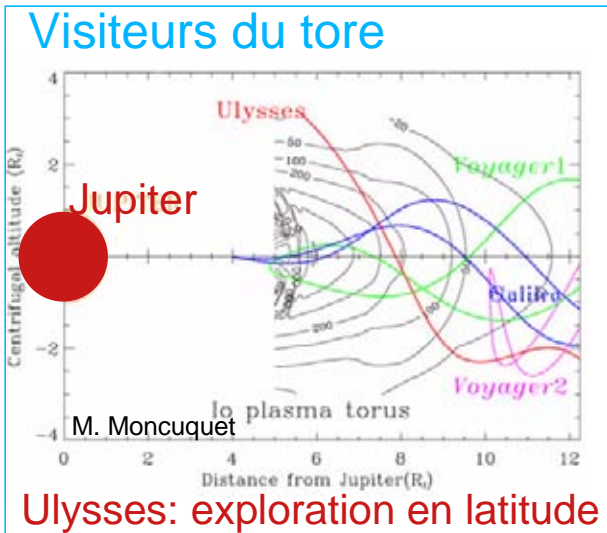
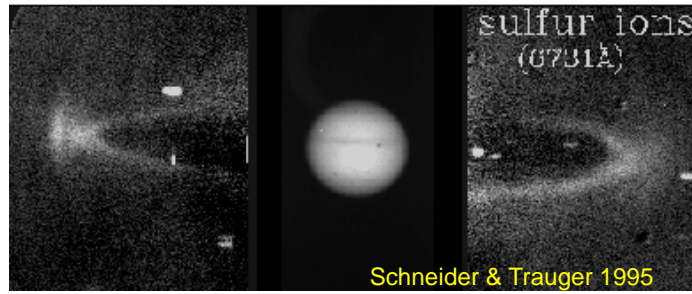
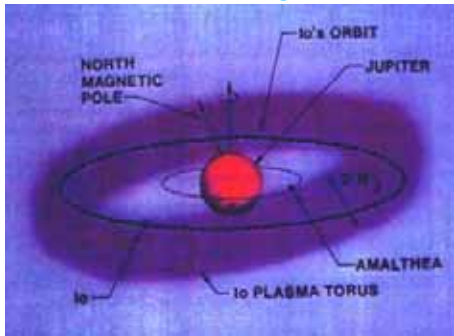
$R \sim 1800$ km, r from Jupiter $\sim 4 \cdot 10^5$ km

\Rightarrow Volcanisme



Ra Patera (Galileo-NASA)

Particules \mapsto ionisation \mapsto pick-up par champ magnétique
 \mapsto Tore de plasma



Champ électrique de polarisation \Rightarrow potentiel total Φ_{tot}

\rightarrow Maxwellienne

$$f_0(v) \propto \exp(-mv^2/2k_B T) \quad \Rightarrow \quad f(v,z) = f_0(v) \exp[-\Phi_{tot}(z)/k_B T]$$

\rightarrow Kappa

$$f_0(v) \propto \left[1 + \frac{mv^2/2}{(\kappa-3/2)k_B T_0} \right]^{-(\kappa+1)} \quad f(v,z) \propto \left[1 + \frac{mv^2/2 + \Phi_{tot}(z)}{(\kappa-3/2)k_B T_0} \right]^{-(\kappa+1)}$$

$$\Rightarrow f(v,z) = t^{-(\kappa+1)} f_0(vt^{-1/2})$$

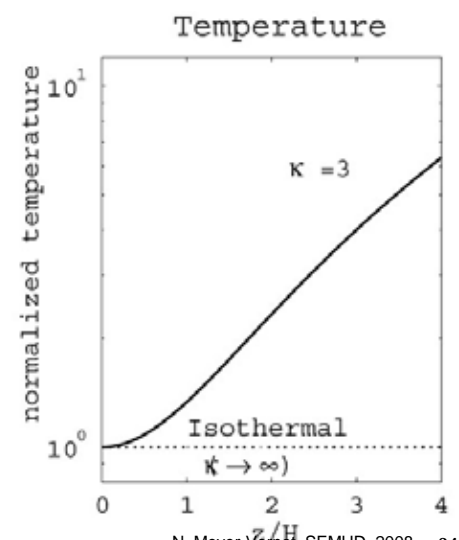
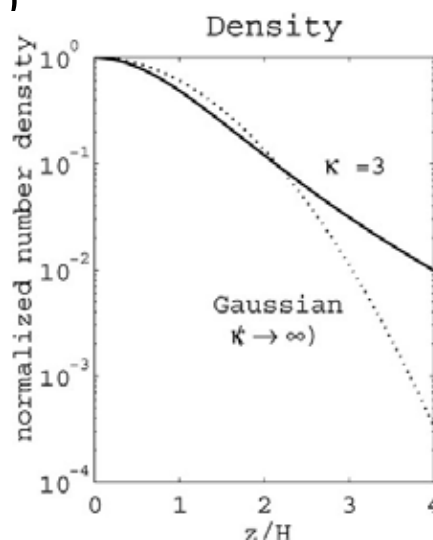
$$t = 1 + \frac{\Phi_{tot}(z)}{(\kappa-3/2)k_B T_0}$$

$$\Rightarrow T \propto t \propto n^{-1/(\kappa-1/2)}$$

Note: anisotropic Kappa : T_{\parallel} increases with altitude

(Moncuquet et al., 2002)

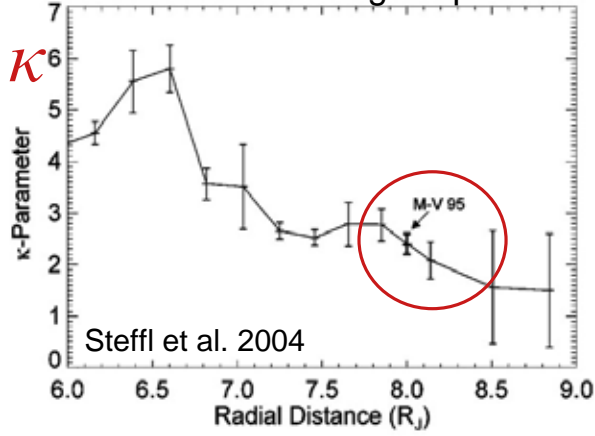
Sum of several maxwellians : T increases with altitude (M-V et al., 1995)



➡ polytrope law $p \propto n^\gamma$
 $\gamma = 1 - 1/(\kappa - 1/2)$

Confirmation par mesures spectres UV

Cassini UltraViolet Imager Spectrometer



Hubble Space Telescope Imaging Spectrograph

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